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Error Propagation of the Computed Orbital Elements of Selected Near-Earth Asteroids

I. Włodarczyk

Astronomical Observatory of the Chorzów Planetarium, WPKiW, 41-500 Chorzów, Poland
e-mail: astrobit@ka.onet.pl, irek@planetarium.chorzow.net.pl

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ABSTRACT

Computed orbital elements of asteroids contain errors depending on the errors of observations. In accordance with the procedure described by Sitarski (1998) we can find randomly selected sets of orbital elements which reasonably represent all observations with fixed mean *rms* residual. In this way we can obtain the error ellipse of the initial orbital elements, and that of the predicted ones. By integrating equations of motion of these computed clones we can obtain a time evolution of changes of the shape of the torus, inside which all the orbits of the clones exist. The time evolution of the configuration of the torus and its size are connected with the asteroid position inside this torus. The larger is the torus the more difficult it is to find the position of the asteroid. The shape of the torus and its time evolution depend mainly on the kind of the asteroid's orbit. If the orbit is more chaotic, then changes of the torus shape are more rapid and the size of the torus is larger.

Close approaches of asteroids to planets are the main source of the chaotic motion. This is particularly important in computing their close approaches to Earth. The distances between the minor planet on the nominal orbit and the virtual minor planets around the nominal orbit can attain considerable values. In this work we computed the time necessary for the values of the mean distances of the clones to achieve the dimensions of the Earth radius. In this respect, we investigated the motion of the known earlier asteroids 433 Eros and 1943 Anteros, and the recently discovered minor planets 99942 Apophis (2004 MN4) and 2004 VD17 – the most dangerous to the Earth, according to the Impact Risk Page of NASA (*http://neo.jpl.nasa.gov/risk/*).

It appears that time-span after which dimensions of the torus attain well defined values are strongly correlated with the stability time and they are also connected with frequent and close approaches to the planets. Furthermore, it was investigated whether the computed orbital elements of the asteroids for the epoch of the beginning, middle or end of the observation, influence the behavior of the asteroids. Also the propagation of the region of uncertainty of asteroid position was computed. This can simplify the computing of close approaches of these asteroids to the Earth and the impact risk assessment.

Key words: *Minor planets, asteroids – Celestial mechanics – Ephemerides*

1. Introduction

Over the last decade, thanks to the rapid development of computing power, it has become possible to use statistical computation of the orbital elements (Bowell *et al.* 2002). This new method takes into account the existence of observational errors. The knowledge of the propagation of errors of the computed orbital elements is important in computing the ephemerides of asteroids, in particular in search for the lost asteroids, in linking of newly discovered asteroids with the lost ones or in precovery and recovery of asteroids as in the case of asteroid (719) Albert (Bowell *et al.* 1993). The propagation of uncertainty of the computed orbital elements is important in the prediction of close approaches to planets, in computing of the possible future impacts of the asteroids with Earth and in impact risk assessment (Milani *et al.* 2002). In all studies the problem of reliability of the obtained results appears. First of all, the errors of computed orbital elements and of the exponential divergence of nearby orbits limit the precise prediction of asteroid motion (Włodarczyk 2001). For instance the prediction of the close approaches of 4179 Toutatis to the Earth is uncertain beyond 100 years from the present time (Whipple and Shelus 1993) or further than 300 years (Sitarski 1998). The main source of this chaotic motion of asteroids are close approaches to planets, among them to Earth. Errors of determination of orbital elements from observations, together with the accumulation of errors resulting from numerical integration of equations of motion of minor planets make it practically impossible to predict the behavior of asteroids even within several decades. In this work we want to show how far in the future we can predict, within acceptable error, the motions of the asteroids. Also our aim is to investigate the parameters which affect the propagation of the computed orbital elements of selected asteroids.

2. Computational Methods

For the prediction of asteroid motion, one must first compute their orbital elements with their associated errors. We can obtain the nominal orbit of any asteroid by the least square correction based mainly on astrometric observations. This nominal orbit will not fit all the observations perfectly, and therefore the residuals will not be zero. The resulting orbit fits all the observations within their expected accuracies, about 1 arc second for optical observations. We may change the orbital elements at random in the range of their mean errors. In this way we can obtain a set of orbits ("clones") which may still fit the observations well and in which the asteroid can move. All these orbits lie within the so called uncertainty region about the nominal orbit. The equations of motion of the clones of the selected asteroids have been numerically integrated 20 000 years forwards by using Mercury Integrator Package v. 6.0 of J. Chambers (Chambers 1999).

The starting orbital elements of the selected clones of asteroids for the epoch of the beginning, middle and the end of the observational arc were computed by Grzegorz Sitarski from the Space Research Center of the Polish Academy of Sciences in Warsaw.

Planetary coordinates were computed using the Warsaw ephemeris of the Solar System: DE405/WAW (Sitarski 2002). The barycentric positions of the planets were adapted to the heliocentric ones by the author. The perturbations of all planets, from Mercury to Pluto, with the Moon treated separately and those of the four biggest asteroids: Ceres, Pallas, Vesta and Hygiea, were taken into account.

3. Starting Orbital Elements of Selected Asteroids

Four objects were chosen from the near-Earth asteroids: two well-known asteroids (433 Eros, first observations from 1893, and 1943 Anteros, discovered in 1968); and two new close-Earth asteroids (99942 Apophis and 2004 VD17 – both discovered in 2004).

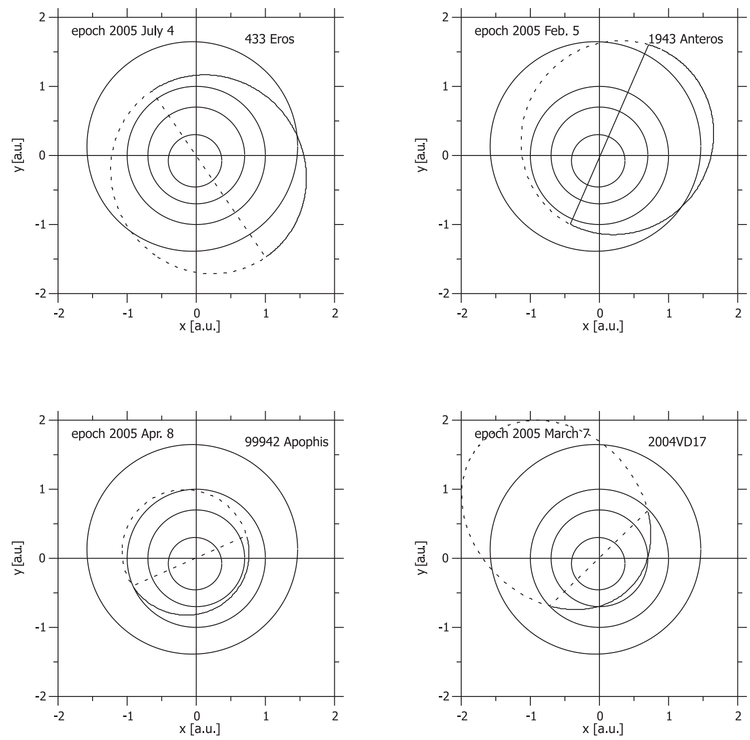

Fig. 1. Nominal orbits of the selected asteroids: 433 Eros, 1943 Anteros, 99942 Apophis and 2004 VD17. The dotted lines indicate the part of the orbit below the ecliptic plane.

Fig. 1 presents the orbits of the selected asteroids on the ecliptic plane. It is clearly seen that orbits of these asteroids approach the Earth and cross the orbits of the other inner planets. All four orbits of the asteroids are well inside the 4 a.u. × 4 a.u. box.

To study the influence of the epoch of the computed orbital elements on the propagation of orbital element errors, three different sets of orbits were chosen – for the 1) beginning, 2) middle and 3) end of the observational arc for each selected asteroid. Table 1 lists these computed orbital elements. For each epoch 200 orbits of the clones (Sitarski 1998) were computed by G. Sitarski.

Table 1

Starting nominal orbital elements of selected asteroids

Nr	M	a [a.u.]	e	ω_{2000}	Ω_{2000}	i_{2000}
433 Eros – 5274 observations from 40 789 days (1893 Oct. 29 – 2005 Jul. 3), $rms = 0\overset{''}{.}82$						
Nominal orbit for the beginning of the observational arc: epoch 1893 Oct. 24.0						
1	309.°15040460	1.45804369260	0.22275909889	177.°49510835	305.°00161542	10.°83462251
Nominal orbit for the middle of the observational arc: epoch 1950 Jan. 19.0						
2	285.°89247718	1.45819437488	0.22300270701	178.°16519743	304.°69561818	10.°83397842
Nominal orbit for the end of the observational arc: epoch 2005 Jul. 04.0						
3	102.°95189106	1.45812119024	0.22277396063	178.°67137896	304.°39109031	10.°82896571
1943 Anteros – 775 observations from 11 654 days (1973 Mar. 10 – 2005 Feb. 4)						
Nominal orbit for the beginning of the observational arc: epoch 1973 Mar. 10.0						
1	330.°43770026	1.43141137237	0.25645967085	337.°89421188	246.°63835595	8.°69534146
Nominal orbit for the middle of the observational arc: epoch 1989 Feb. 21.0						
2	86.°97996637	1.43012411552	0.25608357912	338.°11290413	246.°50234436	8.°70342010
Nominal orbit for the end of the observational arc: epoch 2005 Feb. 5.0						
3	205.°18118558	1.43026762744	0.25592019902	338.°24348827	246.°40866106	8.°70414752
99942 Apophis – 884 observations from 389 days (2004 Mar. 15 – 2005 Apr. 8)						
Nominal orbit for the beginning of the observational arc: epoch 2004 Mar. 15.0						
1	139.°96836059	0.92197289	0.19118625	126.°17021845	204.°57875231	3.°33338897
Nominal orbit for the middle of the observational arc: epoch 2004 Oct. 1.0						
2	2.°62780888	0.92196629	0.19118743	126.°18225693	204.°57563013	3.°33358226
Nominal orbit for the end of the observational arc: epoch 2005 Apr. 8.0						
3	212.°875611054	0.92239674	0.19103240	126.°38279763	204.°47182960	3.°33097679
2004 VD17 – 720 observations from 117 days (2004 Nov. 7 – 2005 Mar. 4), $rms = 0\overset{''}{.}42$						
Nominal orbit for the beginning of the observational arc: epoch 2004 Nov. 07.0						
1	29.°35446892	1.50828865	0.58881659	90.°70558763	224.°25113177	4.°22303575
Nominal orbit for the middle of the observational arc: epoch 2005 Jan. 07.0						
2	61.°83101214	1.50801549	0.58875944	90.°69809952	224.°25044164	4.°22284415
Nominal orbit for the end of the observational arc: epoch 2005 Mar. 07.0						
3	93.°23498004	1.50799076	0.58876064	90.°69693025	224.°25019170	4.°22283177

M is the mean anomaly, a – semimajor axis, e – eccentricity, ω_{2000} – argument of perihelion, Ω_{2000} – longitude of the ascending node, i_{2000} – inclination of the orbit. Orbital elements are referred to the J2000 equator and equinox.

Fig. 2 shows the starting ellipsoid of the errors of the clones of the four selected asteroids computed for the orbital elements for the beginning of the observational

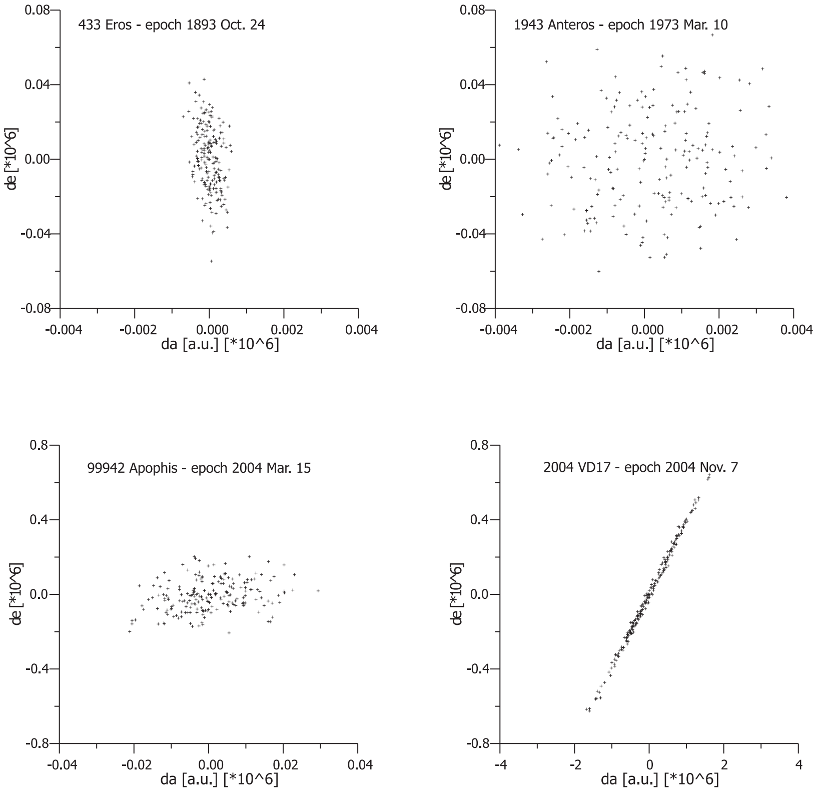

Fig. 2. Starting ellipsoids of the orbital elements errors of minor planets: 433 Eros, 1943 Anteros, 99942 Apophis and 2004 VD17.

arcs. For each asteroid, differences were computed in two orbital elements: in the semimajor axis and in the eccentricity between the nominal orbit and each of the 199 clones. These differences are shown on each axis with a 10^6 enlargement. In Fig. 2 we can see that ellipsoids of error of the two minor planets with the long observational arc – Eros (112 year observational arc) and Anteros (32 year observations), occupy a smaller space than the remaining asteroids with a shorter observational arc. Note that the scales are the same only in the case of Eros and Anteros. For the Apophis asteroid both axes are multiplied by 10 and for 2004 VD17, a new scale for the semimajor axes is specified.

The starting ellipsoid of the error is particularly big for asteroid 2004 VD17 (only 117 days of the observational arc), where differences between semi major axes of the nominal orbit and of the clones reach $2.0 \cdot 10^{-6}$ a.u. *i.e.*, about 0.05 radius of the Earth. It is about 100 times more than in the case of the minor planet Apophis (389-days of the observational arc), over 1 000 times more than for Anteros and over 10 000 times more than for Eros. Here we can clearly see the influence of the length of the observational arc on the dimension of the starting ellipsoid of the errors. We can also call this ellipsoid the region of uncertainty (Milani 2005).

4. Propagation of the Errors of the Computed Orbital Elements

As mentioned before, the determined orbital elements of the selected asteroids contain errors. These errors are a result of the determination process called the best fit solution or the nominal solution. Each asteroid orbital element of the nominal orbit and of the 199 clone orbits were computed. Around the nominal orbit, we obtain a set of orbits which still agree with observations. The orbital elements of the clones differ slightly from the nominal orbit. These clones exist around the nominal orbit with known uncertainty as a result of the determination of the orbit from observations and they are all in the so called uncertainty region about the nominal orbit. The true orbit is inside this region. Therefore, the clones including the nominal orbit are propagated in the future with the use of the suitable integrating procedure.

For each moment of time we can compute the distance between each clone and the asteroid on the nominal orbit and therefore we can compute the mean values of these distances. The time evolution of these mean distances gives us information about the propagation of the clones of the asteroid *i.e.,* how the error of the determined orbital elements of a selected minor planet propagates.

We have shown in Fig. 2 that the error of the determination of the orbital elements depends on the length of the observational arc. The longer the observational arc, the smaller the error of the computed orbital elements leading to a more precise location of the asteroid, and the ellipse of the dispersion of the clones is smaller.

4.1. Different Epochs of the Computed Orbital Elements

To obtain a comparison scale of the results for selected asteroids, the computations were stopped when the dispersion of the mean distances of the clones crossed the size of the Earth's radius. The computations were made for each selected asteroid for different epochs near the beginning, middle and the end of the observational arc. The results are shown in Fig. 3.

We will show that the propagation of the errors of the computed orbital elements depends on the chaotic motion of the asteroid. The closer are the approaches to planets, the more chaotic are the orbits and therefore the differences between the mean anomalies of the asteroids on the neighbor orbits grow rapidly. This suggests that it is practically impossible to predict the behavior of minor planets or comets on the orbit outside this period of time called "the time of stability" (Włodarczyk 2001). However, we find rapid growth of the distances between the clones and the asteroid in the nominal orbit.

Therefore the time when the mean dispersion of the clones achieves the distance of the Earth's radius varies for different asteroids. In the case of Eros, this time is long, over 16 000 years. The orbit of Eros is regular unlike Anteros' chaotic orbit as shown in the cited work. This time of stability of Anteros is only about several hundred years. In Fig. 3 we can see that the clones of Anteros achieve a mean dispersion one Earth radius after 400 years. Similarly, two other minor planets:

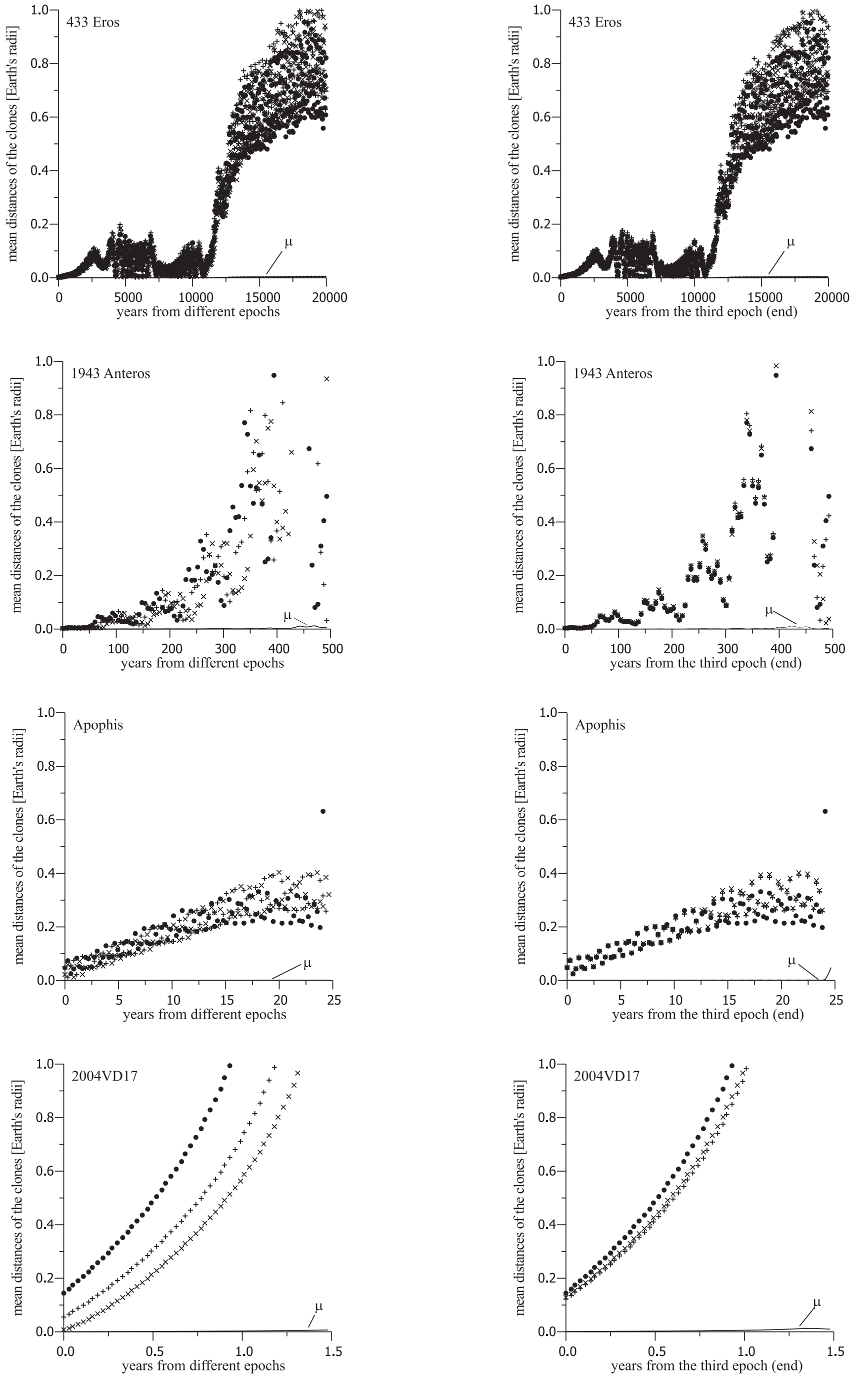

Fig. 3. Propagation of errors of orbital elements computed for the beginning – (x), middle – (+) and the end (o) of the observational arc of selected minor planets. *Left:* the mean distances of the clones [in the Earth radii] *vs.* years from different epochs; *right: vs.* years from the end of the observational arc. Symbol μ denotes the mean error of the arithmetic mean.

99942 Apophis and 2004VD17 have huge chaotic motion. They achieve a mean dispersion of one Earth radius after only 25 years and one year respectively. Hence difficulties in computing close approaches of these asteroids with the Earth and the impact risk assessment arise.

Moreover, it is evident from Fig. 3 that the propagation of the error of the determination of the orbital elements computed for the beginning, the middle and the end of the observational arc of selected asteroids gave similar results. Especially if we start from the epoch of the end of the observational arc (the right sides in Fig. 3).

However, the kind of motion of the asteroid – regular or chaotic – and the length of the observational arc mentioned above are of greater importance. To check the computational results of the propagation of the clones in Fig. 3, the mean error of the arithmetic mean is shown. According to Brouwer and Clemence (1961) if we denote the arithmetic mean of the x_j by $\bar{x}$ then $\nu_j = x_j - \bar{x}$. Hence the mean error of the arithmetic mean, μ

$$\mu = \sqrt{\frac{[\nu^2]}{n(n-1)}} \tag{1}$$

where $[\nu^2]$ is the sum of ν^2. Here, arithmetic mean is the mean of the distances between asteroids on the orbit of the clones and the asteroid on the nominal orbit.

Table 2

433 Eros – results of the forward and backward integration of the nominal orbit of Eros

	M	a [a.u.]	e	ω_{2000}	Ω_{2000}	i_{2000}
Nominal orbit for the beginning of the observational arc: epoch 1893 Oct. 24.0 (JD 2,412,760.5)						
start	309.°1504046	1.4580436926	0.22275909889	177.°49510835	305.°00161542	10.°83462251
10 000 years	309.°1479904	1.4580436917	0.22275909967	177.°49510835	305.°00161576	10.°83462249
20 000 years	309.°1463082	1.4580436913	0.22275910024	177.°49510835	305.°00161600	10.°83462248

M is the mean anomaly, a – semimajor axis, e – eccentricity, ω_{2000} – argument of perihelion, Ω_{2000} – longitude of the ascending node, i_{2000} – inclination of the orbit. Orbital elements refer to the J2000 equator and equinox.

To estimate the error of the orbital elements of Eros, forward and then backward integrations of the equations of motion were made. The equations of motion of the nominal orbit of Eros were integrated 10 000 years and 20 000 years forwards and then backwards to the starting epoch. The results given in Table 2 show that the errors of the integrations of the equations of motion are quite small. Only in one orbital element *i.e.*, in the mean anomaly, the difference between the starting value and the one obtained after forward and backward integration achieves about 0.°0024 and 0.°0041 in 10 000 years and 20 000 years integration, respectively. The errors of the other computed orbital elements are much smaller. Hence the computed results of integration of equations of motion of Eros and other selected asteroids can be applied in this work for the presented time spans.

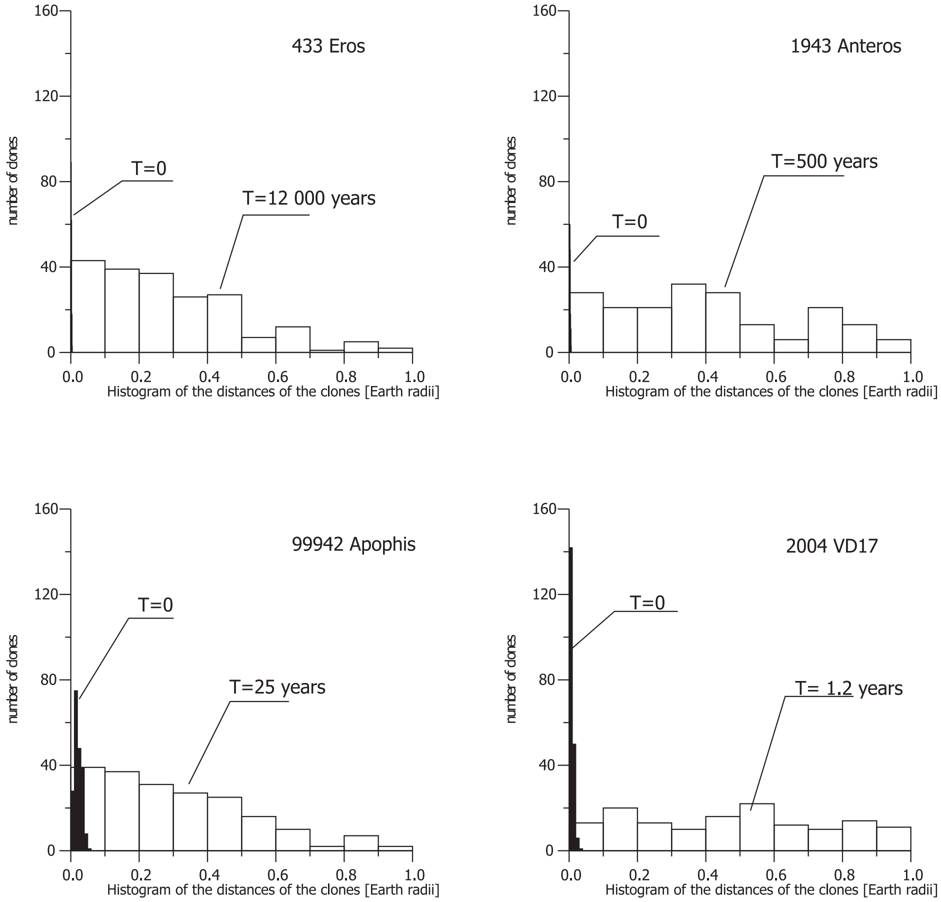

Fig. 4. Histograms of the temporary distances between clones and the nominal orbit in the Earth radii.

Fig. 4 shows the non-averaged distances between the minor planet on the nominal orbit and the clones for the beginning of the time of integration and for the time span where the clones achieve a mean dispersion equal the radius of the Earth. Therefore it may happen that in the time span where the mean distances of the clones do not exceed the radius of the Earth, some temporary distances of the clones from the nominal orbit attain values greater than the radius of the Earth. Thus in the time span of 12 000 years all the clones of the minor planet 433 Eros are situated inside the torus with the size equal to one radius of the Earth. Whereas after 16 000 years, only 128 clones out of 199 remain inside the torus. The histogram of the clones of Eros in temporary distances from the nominal orbit are shown in Table 3.

Table 3

433 Eros – time evolution of the temporary distances of the clones from the nominal orbit

time [kyrs]	Number of clones in the intervals of the distance [a.u.]									
	0– –0.1	0.1– –0.2	0.2– –0.3	0.3– –0.4	0.4– –0.5	0.5– –0.6	0.6– –0.7	0.7– –0.8	0.8– –0.9	0.9– –1.0	sum
start	199										199
1	199										199
2	193	6									199
5	121	62	16								199
12	43	39	37	26	27	7	12	1	5	2	199
16	13	13	15	12	17	11	10	14	15	8	128

In the case of 1943 Anteros, after 500 years, 10 orbits of the clones are outside the sphere of a radius equal one Earth radius. The most distant clone of Anteros is situated 1.6 Earth radius from the asteroid on the nominal orbit. Whereas 3 clones of the asteroid 99942 Apophis leave the one-Earth radius region after 25 years. Their distances from the nominal orbit slightly exceed the radius of the Earth. The clones which leave the one-Earth radius most quickly are the ones of the asteroid 2004VD17. It is worth noting that out of 200 clones of Apophis, only 2/3 remain after 1.2 years. It is evident that histograms of the starting distances of the clones from the asteroid on the nominal orbit are significantly narrower for Eros and Anteros relative to 2004 VD17 and Apophis.

In general, clones hold closer to the nominal orbit, except for 2004VD17, when their distribution is more uniform. The time evolutions of the space density of the possible orbit are almost the same in the sphere. Hence the probability of finding the real orbit inside the sphere is identical for different distances from the nominal orbit.

Fig. 5 shows that the rapid growth of the dispersion of the Eros clones after 4 000 years results from the frequent close approaches to Mars, and in the case of Anteros, the close approaches to Earth. Similarly the cause of dispersion of clones of the asteroid 99942 Apophis are close approaches to Earth in 2029. We see rapid jumps in size of its dispersion. For instance, after 25.2 years from the starting epoch of the orbital elements, the mean dispersion of the clones of Apophis reach 35 radii, and after 40 years about 20 000 Earth radii. For asteroid 2004 VD17, the rapid divergence of the clones result from close approaches to Venus as shown in Fig. 5.

Fig. 6 presents computed times of stability for four selected asteroids. To compare only the influence of the chaotic motion on the orbital evolution of the as-

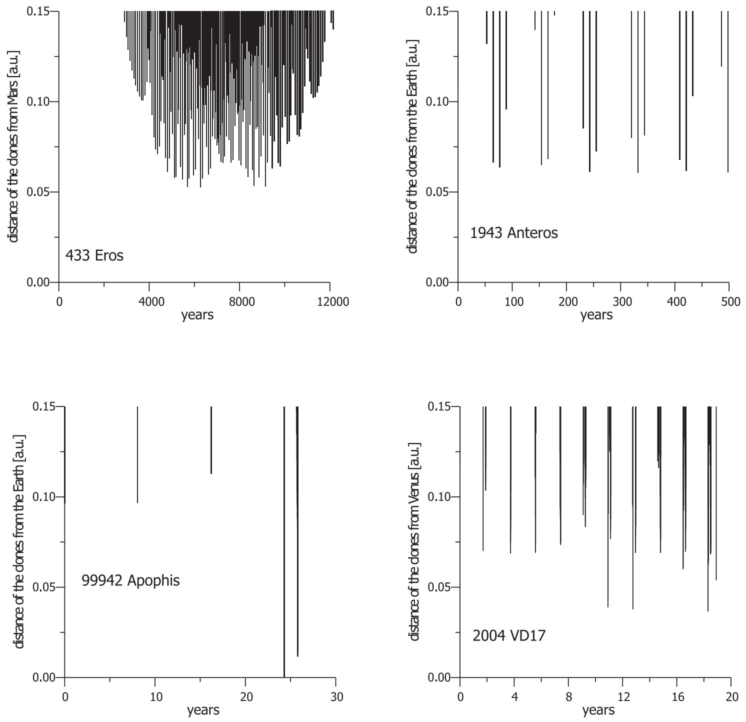

Fig. 5. Close approaches to planets.

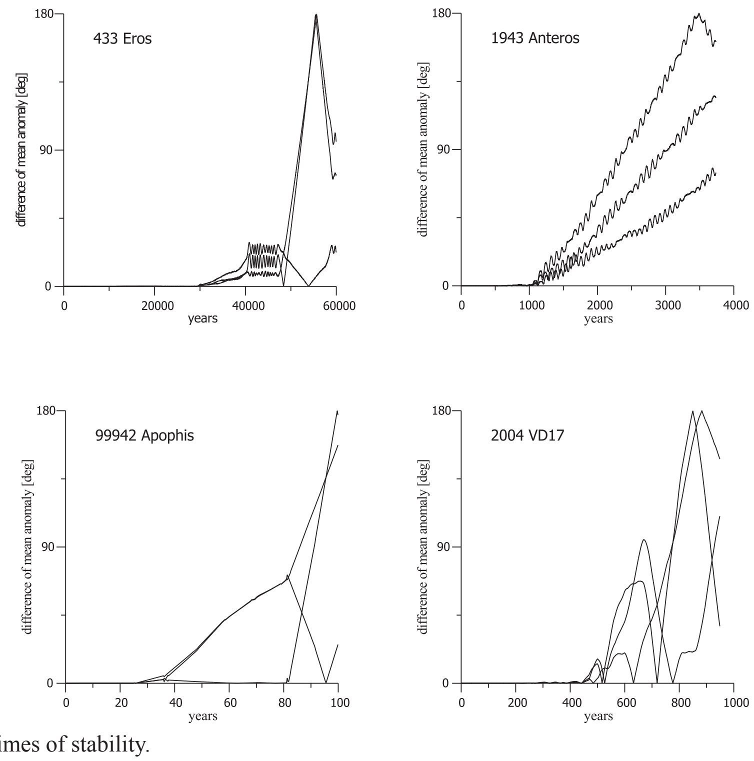

Fig. 6. Times of stability.

teroids, three starting orbits were chosen: the nominal orbit computed for the beginning of the observational arc and the two clones – computed by adding and subtracting only the small value $da = 10^{-8}$ a.u. (1.5 km) to the semimajor axis of the nominal orbit. The other values of the orbital elements remained the same. Then the motion of asteroids on these orbits were computed. If the difference in mean anomaly of neighboring orbits reached 180 degrees, then the computations were stopped. The time of stability is the time after which we observe rapid growth of the differences in the mean anomaly of the neighboring clones. This sudden growth of observed differences is connected with the close approaches to planets, and as a consequence, it points to the chaotic motion of the asteroid. As can be seen, there is a clear relationship between the time of stability in Fig. 6 and the dispersion of the clones in Fig. 3.

4.2. Problem of the Length of the Observational Arc – 99942 Apophis

Propagation of the clones dependence on the length of the observational arc was studied using the observations of the asteroid Apophis. In Fig. 7 we can see that the dispersion of the clones whose orbital elements were determined by the first half of the observational arc are almost two times bigger than if the whole observation arc were used.

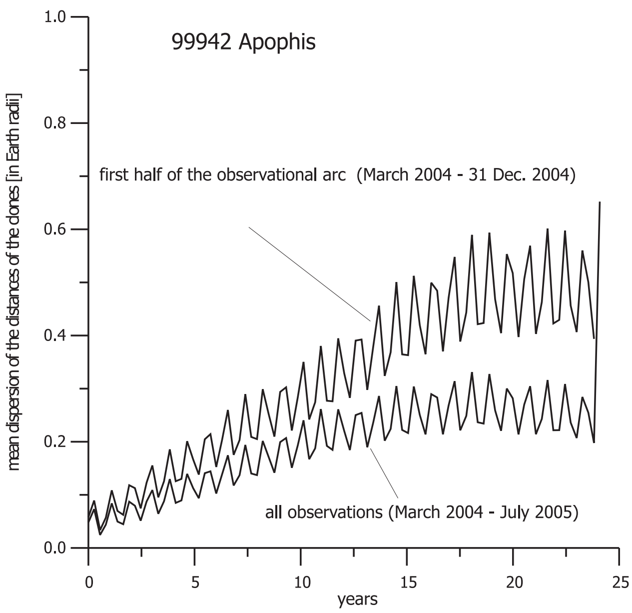

Fig. 7. 99942 Apophis – the different observational arcs

Also the influence of the length of the observational arc for the other asteroids (see Fig. 3) shows that the longer the observational arc, the narrower the sphere of the propagation of clones, and therefore the asteroid is precisely placed in the phase space. The orbits of Eros and Anteros have longer observational arcs than Apophis and 2004 VD17, and hence the former ones preserve their precision for a longer time span.

Table 4

99942 Apophis – orbits computed using different observational arcs

Nr	M	a [a.u.]	e	ω_{2000}	Ω_{2000}	i_{2000}
99942 Apophis – 884 observations from 389 days (2004 Mar. 15 – 2005 Apr. 8), $rms = 0\overset{''}{.}46$						
Orbit for the epoch 2005 Apr. 8.0						
1	$212\overset{\circ}{.}875611054$	0.92239674	0.19103240	$126\overset{\circ}{.}38279763$	$204\overset{\circ}{.}47182960$	$3\overset{\circ}{.}33097679$
99942 Apophis – 329 observations (2004 Mar. 15 – 2004 Dec. 31)						
Orbit for the epoch 2005 Apr. 8.0						
2	$212\overset{\circ}{.}87599812$	0.92239616	0.19103212	$126\overset{\circ}{.}38193365$	$204\overset{\circ}{.}47251401$	$3\overset{\circ}{.}33096826$

4.3. The Problem Eros – Anteros

As mentioned above, Eros and Anteros have similar orbits, but the propagation of their clones is completely different. What is the cause of the different behavior of these asteroids? To eliminate the influence of the length of the observational arc, and the number of asteroid observations, a selection of a subset of their observations was made. The observations of asteroid Eros were chosen only from the time-span of the period of the observations of Anteros: 1973 March, 14.75320 to 2005 March, 08.67650. The first observations of the asteroid Anteros from 1968 were rejected due to the great residuals $O-C$ in the process of determination of the orbit. In this period there were about 3 000 observations of Eros, about four times as many as for Anteros. Therefore, every fourth observation was selected from these observations of Eros. Thus, 763 observations of Eros were chosen, a number comparable to the 775 observations of Anteros. After this procedure, we obtained a set of similarly spaced in time observations of Eros and Anteros. Next, we computed the nominal orbits of Eros and Anteros and of their 199 clones. The results of integration of the equations of motion of the clones are presented in Fig. 8. From these computations it is clear that the only cause of differences in the propagation of clones of Eros and Anteros are the ones connected with the close approaches to planets as shown in Fig. 5. Eros has frequent close approaches to Mars whereas Anteros has close approaches to the Earth. Both asteroids attain similar distances to the Earth and to Mars, of about 0.05 a.u. to 0.1 a.u. However, it is worth noting that Anteros has close approaches to the Earth which has a greater mass than that of Mars. Moreover the orbit of Anteros is more tangential to the orbit of the Earth (the inclination of orbit to the ecliptic plane, $i = 8\overset{\circ}{.}7$) than the orbit of Eros ($i = 10\overset{\circ}{.}8$). This is an additional cause of the different behavior of these asteroids.

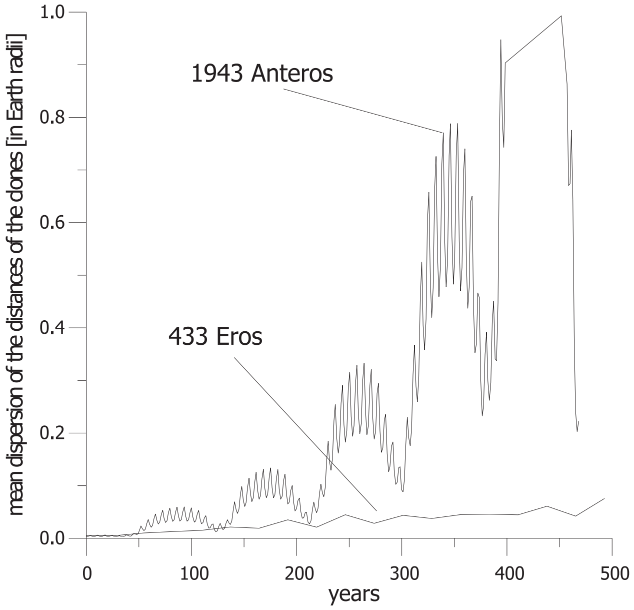

Fig. 8. Eros – observational arc trimmed to the Anteros one.

Table 5

Starting nominal orbital elements of Eros and Anteros normalized to observational arc of Anteros

Nr	M	a [a.u.]	e	ω_{2000}	Ω_{2000}	i_{2000}
	Orbit of Anteros for the end of the observations arc: epoch 2005 Feb. 5.0					
1	205°.18118558	1.43026762744	0.25592019902	338°.24348827	246°.40866106	8°.70414752
	Orbit of Eros: epoch 2005 Mar. 9.0					
1	37°.45013740	1.45818348264	0.22277025076	178°.67433495	304°.39910374	10°.82906384

4.4. The Perihelion–Aphelion Dependence

Observations of Eros made in perihelion and in aphelion were examined for the propagation of errors of the computed orbital elements. From all 5 409 observations of Eros covering Oct. 29, 1893 to Aug. 25, 2005 using criterion $170^\circ < M < 190^\circ$ for the arc of the aphelion and $350^\circ < M < 10^\circ$ for the perihelion, only those satisfying these criteria were chosen. Hence, only 161 observations from the aphelia and 580 observations from perihelia were collected. Table 6 lists the starting orbital elements of the nominal orbit of Eros computed from all 5 409 observations, from 161 observations in the aphelia, from 611 observations in the perihelia and from 153 selected observations in its perihelia. Using every fourth observation in perihelion we retrieved almost the same number of observations as in aphelion, 161 and 153, respectively.

Table 6

Starting orbital elements of the nominal orbit of the Eros computed from all the observations, from the observations made near aphelia and near perihelia of the Eros orbit

Nr	M	a [a.u.]	e	ω_{2000}	Ω_{2000}	i_{2000}
The epoch 2006 May 25.0						
433 Eros – all observations, $rms = 0''.82$						
1	$284^{\circ}.91299365$	1.45814095978	0.22273297297	$178^{\circ}.64960591$	$304^{\circ}.38769163$	$10^{\circ}.82869622$
433 Eros – observations near the aphelia, $rms = 0''.72$						
2	$284^{\circ}.91401646$	1.45814097917	0.22273657982	$178^{\circ}.64897842$	$304^{\circ}.38759235$	$10^{\circ}.82871062$
433 Eros – observations near the perihelia, $rms = 0''.82$						
3	$284^{\circ}.91309353$	1.45814095954	0.22273288767	$178^{\circ}.64945961$	$304^{\circ}.38768460$	$10^{\circ}.82869079$
433 Eros – selected observations near the perihelia, $rms = 0''.77$						
4	$284^{\circ}.91309550$	1.45814095959	0.22273287866	$178^{\circ}.649455535$	$304^{\circ}.38768455$	$10^{\circ}.82869165$

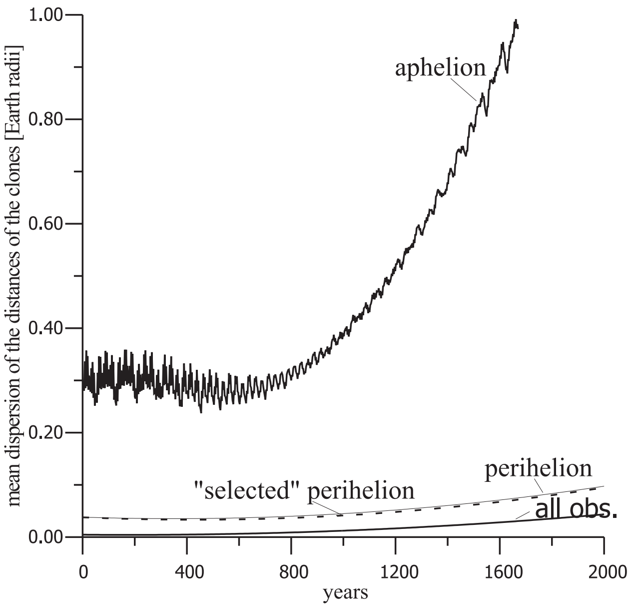

Fig. 9. Eros: problem perihelion–aphelion.

It is clear from Fig. 9 that the propagation of the orbital elements errors made from observations in aphelia are about ten times more divergent than those in perihelia and about thirty times more divergent than those using all the observations. The orbit computed using only the observations from the perihelia is more precise compared to orbits computed only from aphelion observations. This is due to the smaller distance of the asteroid to the Earth in perihelia: about 0.80 a.u. in aphelion and 0.14 a.u. in perihelion, respectively. Moreover in the vicinity of opposition and in aphelion of Eros, the distances between the asteroid and Earth can be significantly greater than 0.80 a.u., in contrast to these distances during observations made in opposition and in perihelion.

There is almost no difference between the time evolution of the mean dispersion of the clones computed from all observations in perihelion and from the selected ones (dashed line in Fig. 9). The number of observations does not influence the propagation of errors.

The precision of the computed orbital elements of Eros from all observation is impressive. All the propagated clones remain well within the ellipsoid of 0.05 Earth radii (about 300 km) over the next 2 000 years!

5. Importance of the Distribution of Errors of Computed Orbits for the Observer

For observers of asteroids and comets, the precision of the computed orbital elements is important. In particular they want to know in which region of the sky the given object should be found. This naturally depends on the accuracy of the orbit. Each of the computed clones of the asteroid is located in a certain distance from the asteroid on the nominal orbit. In the sky, the clones are located in a given region which we can call the ellipse of dispersion or the uncertainty region (Milani 2002), or the confidence boundary described in Milani (NEODYS) www site: *http://131.114.72.13/cgi-bin/neodys/neoibo* where we can also find the observation predictions for NEOs. Milani's computations have been made with the use of the probability of the determination of the orbital elements.

Fig. 10 presents the region in the sky occupied by clones for a given epoch of orbital elements. The size of this region depends on the precision of the orbit. Each point in Fig. 10 indicates the difference in arcsecs in right ascension and declination between the position of the asteroid on the nominal orbit and on one of the 199 clones. As Fig. 10 shows the orbits of the Eros and Anteros are precisely computed, and the observer can search for these asteroids in the radius of about $rms = 0\overset{''}{.}05$ around the asteroids on the nominal orbit. In the case of Apophis the radius of the confidence boundary is three times greater and for 2004 VD17 the observer has to search for the asteroid in the region of the size of $rms = 0\overset{''}{.}60$.

However the confidence boundary growth with time. It was shown in Fig. 11 that after only 100 days from the starting epoch of the orbital elements, the size of this region for Apophis and for 2004VD17 is almost three times bigger. For the poorly known orbit of the asteroid 2004 VD17, the observer should search for the asteroid in the region of the size of $rms = 1\overset{''}{.}50$. But in the case of orbits of Eros and Anteros, the confidence boundary is almost the same. It is worth noting that the size of the confidence region in right ascension and declination depends on the distance of the asteroid.

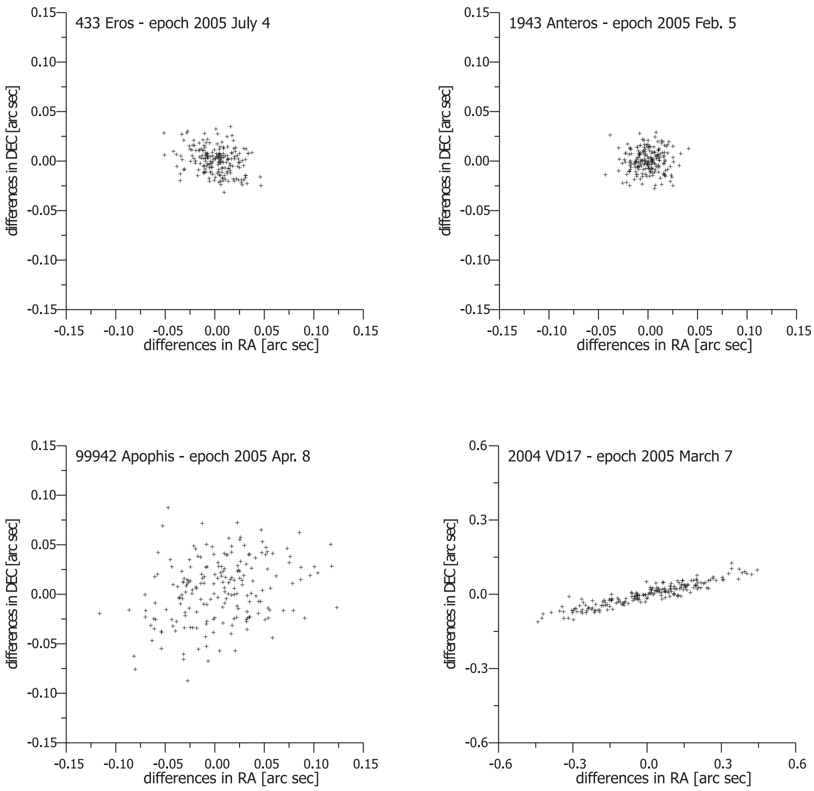

Fig. 10. Distribution of errors of computed orbits in the sky plane for the starting epoch. The starting confidence boundary.

For longer time spans, the region of the uncertainty grows considerably. Hence we can see how important the precision of the computation of orbital elements for a given epoch is. The uncertainty in the determination of the orbital elements of the asteroid affects the uncertainty of the computation of the asteroid ephemeris.

6. Summary

Studying the motion of selected asteroids, we see that the error of propagation of the determination of the orbital elements is particularly important in computations of precise ephemerides of the asteroids. This is essential in computing the ephemerides of asteroids with a short observational arc and with a small number of observations when the uncertainties become large.

Moreover, it becomes evident that error propagation of asteroid clones depends on the kind of the orbit. Asteroid orbits become much more chaotic following close approaches to planets, and thus the propagation of the errors of the computed orbital elements is less predictable.

In the case of the near-Earth asteroids, the distances to the Earth during ob-

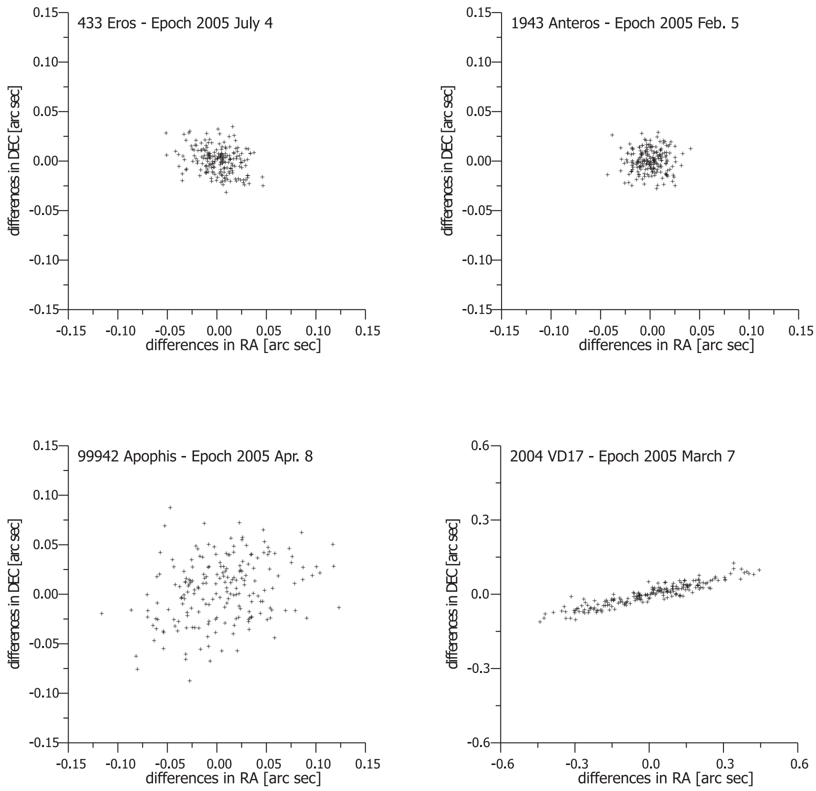

Fig. 11. Distribution of the error of the computed orbit in the sky plane after 100 days from the starting epoch. Confidence boundary after 100 days.

servation periods has a great influence on the precision of these orbits. When the asteroid is located closer to Earth its observations are more precise than those made at longer distances.

It follows from this work that the knowledge of the propagation errors can help in studying the behavior of asteroid in the future. The knowledge of the propagation of computed orbital element errors of asteroids can be the basis for risk assessment of potentially hazardous asteroids for the Earth similar to the one presented on the NASA risk page (*http://neo.jpl.nasa.gov/risk/*).

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REFERENCES

Bowell, E., *et al.* 1993, *Bull. Am. Astron. Soc.*, **25**, 1118.
Bowell, E., *et al.* 2002, in: "Asteroids III", p. 27.
Brouwer, D., and Clemence, G.M. 1961, in: "Methods of Celestial Mechanics", Ed. Academic Press, New York and London, p. 215.
Chambers, J.E. 1999, *MNRAS*, **304**, 793.
Kristensen, L.K. 1992, *Astron. Astrophys.*, **262**, 606.
Milani, A., *et al.* 2002, in: "Asteroids III", p. 55.
Milani, A., *et al.* 2005, *Astron. Astrophys.*, **431**, 729.
Sitarski, G. 1998, *Acta Astron.*, **48**, 547.
Sitarski, G. 2002, *Acta Astron.*, **52**, 471.
Whipple, A.L., and Shelus, P.J. 1993, *Icarus*, **105**, 408.
Włodarczyk, I. 2001, *Acta Astron.*, **51**, 357.